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An investigation of the noticing skills of mathematics teachers to their students' misconceptions in written responses[1]

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Abstract

The study aimed to determine the noticing skills of mathematics teachers to their students' misconceptions in written responses. The study adopted case study design, one of the qualitative research methods. The participants of the study comprised 20 volunteer middle school mathematics teachers from 9 different cities and 16 different schools that were selected via convenience sampling method in 2017-2018 school year in Turkey. The schools were determined based on the examination called 'Nationwide Standardized Level Determination Exam' as high, medium and low achievement school levels. Considering the idea that teaching experience could affect noticing, special attention was paid while choosing teachers with different years of teaching experience. The data were collected using a "diagnostic test of misconceptions" developed by the researchers, and video-recorded clinical interviews with teachers. The diagnostic test included 13 open-ended questions, and was administered to 497 students (7th grades) by their own teachers. During the data analysis, first, the students' answers were examined in the context of misconceptions in order to identify the students who had misconceptions in their written responses. Then the participant teachers were interviewed considering their students' answers containing misconceptions. Finally, both types of data were analyzed via content analysis method. The findings showed that there were many students, who had misconceptions in their written responses, and the teachers' viewpoints regarding the misconceptions were narrow and their noticing skills to students' misconceptions were low.

Keywords: Noticing skills; mathematics teachers;middle school students; misconceptions; written responses

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1. Introduction

Educational programs are developed with the goal of having students gain some pre-determined objectives (Ertürk, 1997). They also aim to answer the questions; "Why should we teach?", "What should we teach?", "How should we teach?" and "How much have we taught?". Four basic elements to answer these questions are respectively "aim", "content", "education process" and "assessment and evaluation" (Ornstein & Hunkins, 1993). While aim, content and teaching process lead the education programs, assessment and evaluation processes have an important role in determining whether the education program has achieved its purpose (Demirel, 2009). Assessment and evaluation constitute the first stage of the education process that will be started and provide information about the dimension of the education necessary to reach the objectives (Küçükahmet, 2003). According to the assessment and evaluation results, the education program is revised, necessary changes are made and the new education program developed is put into practice. In other words, the assessment and evaluation processes which show the extent to which the expected behavior change occurs in the individual at the end of the education process form an integral part of the education programs (Demirel, 2009). Regarding the assessment and evaluation studies, it may be asserted that the assessments are made countrywide or by teachers. Written exams are assessment and evaluation instruments, which are frequently used by teachers (Nazlıçiçek & Akarsu, 2010). Therefore, it is important for teachers to be able to understand student thoughts from written responses.

1.1. The Importance of Assessment and Evaluation: The Noticing Skills

Determination of students' misconceptions is one of the important criteria in the assessment and evaluation processes (Djanette & Fouad, 2014). As the students' experiences regarding knowledge and the meanings they attribute to concepts in their minds are different, teachers are expected to reveal these differences through assessment and evaluation. Misconceptions are revealed and different meanings contribute to the following educational activities. Teachers should determine and to remove students' misconceptions (Smith, diSessa, & Roschelle, 1993). Students' misconceptions can be determined by interpreting the questions asked by the students or their answers. This requires teachers to be able to interpret student answers well and identify misconceptions. Shulman (1986) expresses that the determination of misconceptions by asking qualified questions, the ability to reveal students' thinking and to treat the misconceptions by analyzing these thoughts constitute a part of pedagogical context knowledge. Teachers who have the skill of noticing students' misconceptions can better manage the education process. This indicates that noticing is a skill, which is expected from teachers.

Considering the teaching profession, it is seen that noticing has a complex and challenging structure (Jacobs, Lamb, & Philipp, 2010; van Es, 2011). Noticing is accepted as an important component of teaching specialization because this skill affects the teaching quality (Schack, Fisher, & Wilhelm, 2017). Teachers who have noticing skills can determine the meaningful mathematical thoughts of their students and make decisions for the next instructional step depending on students' thinking and understanding (Krupa, Huey, Lesseig, Casey, & Monson, 2017). Jacobs et al. (2010) discussed the skills of noticing students' mathematical thinking in 3 related components: a) determining students' strategies of solution, (b) interpreting mathematical understanding of students, and (c) decision making how to feedback to students' understanding. Studies show that teachers and prospective teachers have low-level noticing skills (Jacobs et al., 2010; van Es, 2011; Özdemir-Baki, & Işık, 2018) and training can improve these noticing skills (Jacobs et al., 2010; van Es, 2011; Özdemir-Baki, & Işık, 2018; Roller, 2016). In these studies, the course videos were watched and all the components of noticing that emerged were discussed by interpreting the course process. Regarding the results of the studies, it was seen that teachers' and prospective teachers' skills of noticing students' thinking generally emerged in situations that could be clearly seen in the video and were limited in terms of in-depth determination of student thoughts (Tataroğlu-Taşdan, 2019; van Es & Sherin, 2008; Ulusoy & Çakıroğlu, 2018).

There are also studies focusing on how teachers' and prospective teachers' noticing skills to students vary in different student performances. In these studies, for the noticing of students' mathematical knowledge, teachers and prospective teachers were asked to examine the sample videos and the written answers of the students (Estapa et al., 2018; Goldsmith & Seago, 2011). Estapa et al. (2018) showed that prospective teachers focused more on students in written responses and more on teachers in videos. Goldsmith and Seago (2011) found that teachers working on video analysis caught the students' thoughts at the beginning of the video, and then they focused less on the students' thoughts. They expressed that the students' public thoughts may not be real thoughts. It was seen that teachers working with written answers made more efforts to realize and interpret student understanding since it is difficult to reach the reasons related to solutions in written answers. These results, on the other hand, increased the burden of understanding students' thoughts on video lessons, but revealed the strong effects of the nature of written answers on teachers' perception and interpretation of students' understanding. In the written answers of the students familiar with the teachers, it was observed that the teachers made comments about the students' thinking considering the situation of the student in the classroom (Goldsmith & Seago, 2011). In this context, in professional development, managers should make qualified decisions for the type of performance to be used and should use the classroom practices (teacher activities) carefully to require teachers to pay attention on the mathematically important (Goldsmith & Seago, 2011) elements. It is seen that different performances can be used to achieve different professional developmental goals. In this context, it may be preferable to work with written performances if the aim of the study is related to the skill of noticing students' thoughts. In this study, the skill of noticing students' thoughts was specifically addressed in the context of the second component, which Jacobs et al. (2010) discussed and the teachers' ability to notice students' misconceptions was stressed. Thus, we tried to separate the other noticing components that appeared in the videos and only aimed to focus on noticing students' thoughts. Another reason for this is to reveal the teachers' ability to determine students' thoughts in written exams, one of the goals of which is to determine whether the educational activities have achieved their purpose. Determining student understanding in written answers may give the teachers an idea of future practices and may allow teachers to treat students' misconceptions. In this context, it is thought that the results of this study have potential to illuminate the studies to be conducted on teachers' professional development. The purpose of the study was to determine the noticing skills of middle school mathematics teachers to their students' misconceptions in the written responses. As a consequence, the research question of the study is: "**How are the noticing skills of middle school mathematics teachers to their students' misconceptions in the written responses?"**

2. Method

The study aimed to summarize an existing situation via descriptive methodology. So the study adopted the case study method as a type of qualitative research. Determination of the details that make up a situation, development of possible explanations about a situation, and the evaluation of a situation are all possible through a case study (Gall, Gall & Borg, 2007). In the study, the noticing skills of the participant mathematics teachers about their students' misconceptions in their written responses were investigated, and no intervention was made to the existing situations.

2.1. Participants

In the 2017–2018 school year, during spring semester, 20 middle school mathematics teachers from 9 different cities and 16 different schools participated in this study. Cities and schools were selected via convenience sampling method. Schools were determined based on the results of the "Nationwide Standardized Level Determination Exam" as high, medium and low achievement school levels. The participant teachers were also selected using convenience sampling model considering the teachers' years of teaching experience. The teachers were informed about the study, and the volunteer teachers with different years of teaching experience participated in the study. The participants were coded as T1, T2, T3 etc. Table 1 below shows the distribution of the participants' school level and year of teaching experience.

Table 1. Distribution of the participants' school level and years of teaching experience

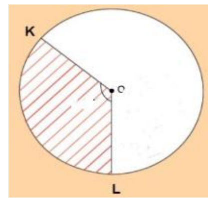
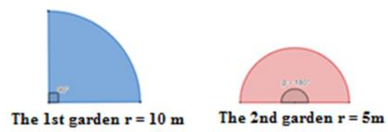
Demographic Features		Participants	f	%
School Level	Low	T1,T4,T13,T16,T18,T20	6	30%
	Medium	T2,T5,T7,T8,T14,T15,T17	7	35%
	High	T3,T6,T9,T10,T11,T12,T19	7	35%
Year of Service	1-5 Years	T1,T4,T5,T7,T8,T12,T13,T15 T16,T17,T18	11	55%
	5+ Years	T2,T3,T6,T9, T10,T11,T14, T19,T20	9	45%

When we compare the percentage of the participants in terms of the level of the schools they work in, it may be asserted that the percentages are close to each other. In Table 1, it can be seen that the participants are classified according to their being experienced and inexperienced, and that the percentages are close to each other. It is thought that this situation will contribute to reflecting on the general situation.

2.2. Data Collection Process

The middle schools' mathematics teachers teach 5-8 graders in Turkey, and one particular teacher may teach different grades in the same school year. The common teaching grade level of the participants was the seventh grade. In accordance with the purpose, a misconception test for the 7th graders was developed by the researchers, considering the curriculum, relevant literature, and researchers' years of teaching experience. While developing the test, the learning outcomes of the 7th graders in the Spring Term introduced by the Ministry of National Education, Turkey [MoNE] (2013) were examined. Afterwards, the misconceptions in the literature regarding these outcomes were determined. The test items were developed in accordance with the common misconceptions in the literature. Expert opinions were obtained about test items to define the misconceptions. As a result, the diagnostic test was finalized with 13 open-ended questions. Learning outcomes, related misconceptions and sample questions are stated in Table 2 below.

Table 2. Examples of the questions in the test

Outcome	Misconception	Example Question
Students calculate the circumference of a circle and perimeter of a sector. Students calculate the area of a circle and area of a segment.	When the sector is removed, the area of the shape decreases, so the perimeter decreases, too.	As seen in the image, Ahmet wants to cut a sector. When he cut the marked sector;  How does the perimeter change? How does the area change? Explain what you think.
Students calculate the area of a circle and area of a segment.	As the central angle increases, the area of the sector increases.	 Uncle Ali is going to plant flowers in his two gardens seen below. What do you think about in which garden the flowers are going to be planted in a bigger area? What did you think while answering the question?
Student calculates one quantity as a percentage of another quantity.	The increase or decrease in an amount is same as increase or decrease in percentage.	 The pine tree grows its length from 1.5 m to 2 m in a year. The apple tree grows its length from 2 m to 2.5 m in a year. Which tree is growing faster? How did you decide?
Students explain whether the two variables are directly proportion and inversely proportional.	Variables are multiplied in the directly proportion; variables are divided in the inversely proportion.	$\frac{4}{A}=\frac{5}{B}$ A.B=4 $\frac{A}{B}=2$ Which of the equations above are directly proportions or inversely proportions? How did you decide?

The diagnostic test was administered to 497 students (7th grades) by their own teachers. The students were allocated 30 minutes to complete the test. After they complete the test, first, their' answers were examined in the context of misconceptions from researchers. For the analyzing procedure, the correct answers were determined clearly by two researchers.

The answers were categorized as true, partly true, false or no answer. False answers were examined in order to determine whether these answers were due to errors or challenges. Two researchers tried to determine the misconceptions. Regarding the disagreements, the third field expert's opinion was taken. The researchers discussed until they agreed upon the analysis. For each question, students were categorized according to having similar misconceptions. Then, prototype answers were determined for each misconception. Clinical interviews were carried out with the teachers according to their students' prototype answers. During the interviews, the question "What could your student be thinking while giving this answer?" was asked to teachers for each defined student answer. The interviews were conducted individually and they were based on the teachers' own students' answers.

2.3. Data Analysis

Teachers were interviewed considering their students' written answers, which contain misconceptions. Clinical interviews were undertaken and video-recorded by one of the researchers for approximately 40 minutes. The videos were analyzed by two researchers using the categorical analysis technique, which is a type of content analysis. The researchers examined the videos in order to determine the teachers' skill of noticing about students' misconceptions. Each of the researchers explained their ideas regarding teachers' thoughts and they tried to reach a consensus about the answer. The interviews were analyzed by two researchers with decision procedure. The third field expert was conferred with when disagreements occurred. The researchers discussed until they agreed upon the analysis. Thus, analysis reliability was tried to be provided.

During the study, through the question directed to the teachers "What could your student be thinking while giving this answer?", the teachers' answers regarding the students' misconceptions were evaluated as "Notice the misconceptions" and "Do not notice the misconceptions". As the questions in the test aimed to define the misconceptions about different issues, average percentages were determined to evaluate the skills of noticing misconceptions. This was because the purpose of the study was to determine teachers' skills of noticing misconceptions in any subject rather than misconceptions in a specific context.

3. Results

Table 3 below shows the findings on the participants' skills of noticing the students' misconceptions according to each question in the test.

Table 3. Findings on the participants' noticing skills to their students' misconceptions

Questions	Notice the misconceptions		Do not notice the misconceptions	
	f	%	f	%
Question 1	12	60%	8	40%
Question 2	8	40%	12	60%
Question 3	12	60%	8	40%
Question 4	14	70%	6	30%
Question 5	8	40%	12	60%
Question 6	10	50%	10	50%
Question 7	14	70%	6	30%
Question 8	14	70%	6	30%
Question 9	8	40%	12	60%
Question 10	6	30%	14	70%
Question 11	2	10%	18	90%
Question 12	8	40%	12	60%
Question 13	8	40%	12	60%
Average Percentage	9.5	47%	10.5	52%

In Table 3, it can be seen that the teachers' level of noticing misconceptions was 70% for only three questions and it was below 70% for ten questions. The average noticing skill of the teachers was found to be 47%. Student answers with misconceptions directed to teachers belong to the teachers' own students. This shows that 52% of the misconceptions are generally ignored. It seems remarkable that the level of noticing some misconceptions is high and some as low as 10%.

According to Table 3, when the percent value of the teachers' awareness level of the students' misconceptions was averaged, it was seen that the awareness of misconceptions was low. Students' sample answers in the misconceptions test and teachers' comments on these answers are exemplified in the following part. Figure 1 shows the written answer given by a student of T7 to the first question in the test. All questions, answers and interviews are in Turkish. It has been translated into English by researchers.

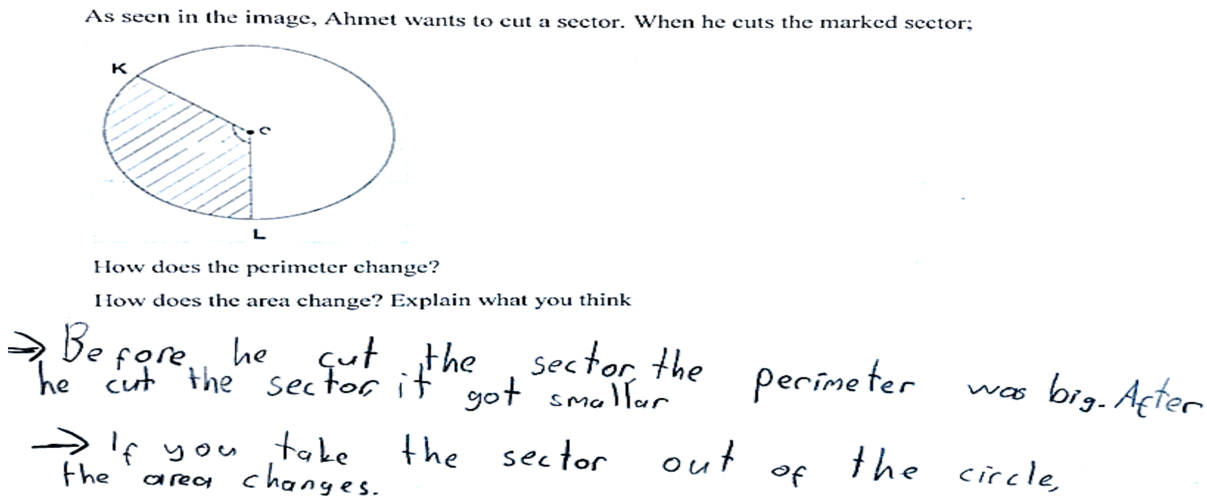


Figure 1. The student answers to the 1st question

The question assesses the learning outcomes of the 7th grade, such as “Students calculate the circumference of a circle and perimeter of a sector” and “Students calculate the area of a circle and area of a segment”. It can be seen that there are some misconceptions about these outcomes. The student’s written answer was shown to his/her teacher (T7), and the teacher was invited to answer the question; “What could your student be thinking while giving this answer?”. T7’s answer to the question was:

> T7: “The student might not have thought of possible changes because she was not able to learn the circumference and area concepts properly.”

It is apparent that the teacher could not understand the student’s perspectives clearly. S/he was not aware of the student’s overgeneralization. However, the student generalized the knowledge that "when the sector was removed the area decreases" to the circle perimeter. S/he had the misconception in the literature: “When the segment is removed, the area of the circle decreases, so the perimeter decreases, too”. 60% of the interviewed teachers could interpret the reason of the students’ misconceptions while 40% of them could not. Figure 2 shows the written answer given by a student of T13 to the 4th question in the test.

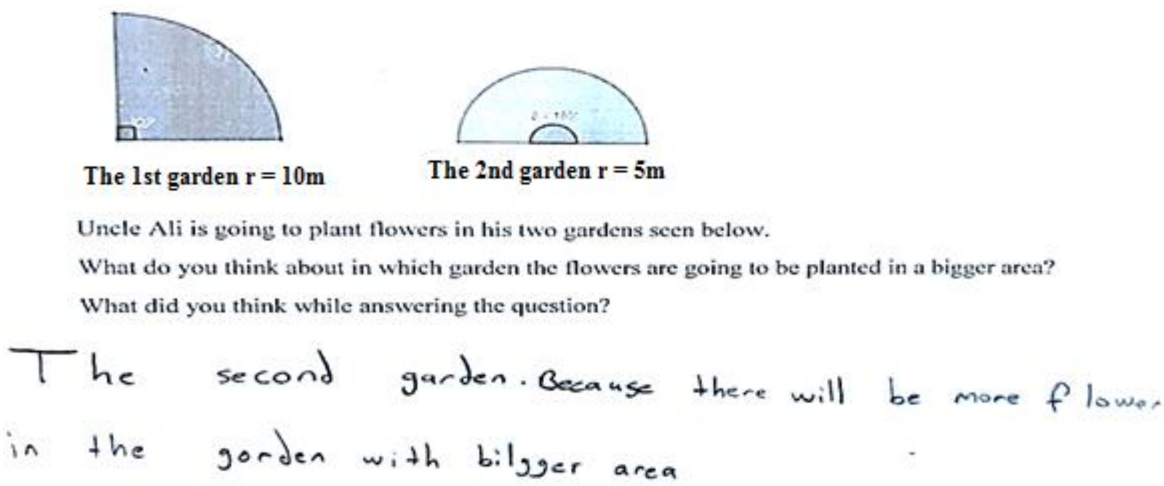


Figure 2. The student answer to the 4th question

The question aimed to assess the learning outcome; "Students calculate the area of a circle and area of a segment". T13's student's answer was shown to T13 and asked "What could your student be thinking while giving this answer?". The answer was:

> T13: S/he made a mistake because s/he paid no attention to their Radius."
>
> T6's student answered with a similar perspective;
>
> T6: "The student made an overgeneralization by thinking that the circle with a wider angle is bigger than the other one."

As is seen, the student has the misconception in the literature: "As the central angle increases, the area of the sector increases". While T13 is not aware of the student's misconception and explains that the wrong answer stems from the mistake, T6 is capable of determining the student's misconception.

70% of the interviewed teachers could interpret the reason of the students' misconceptions while 30% of them could not exactly interpret it and thought that they made a mistake. Figure 3 shows the written answer given by a student of T5 to the 6th question in the test.

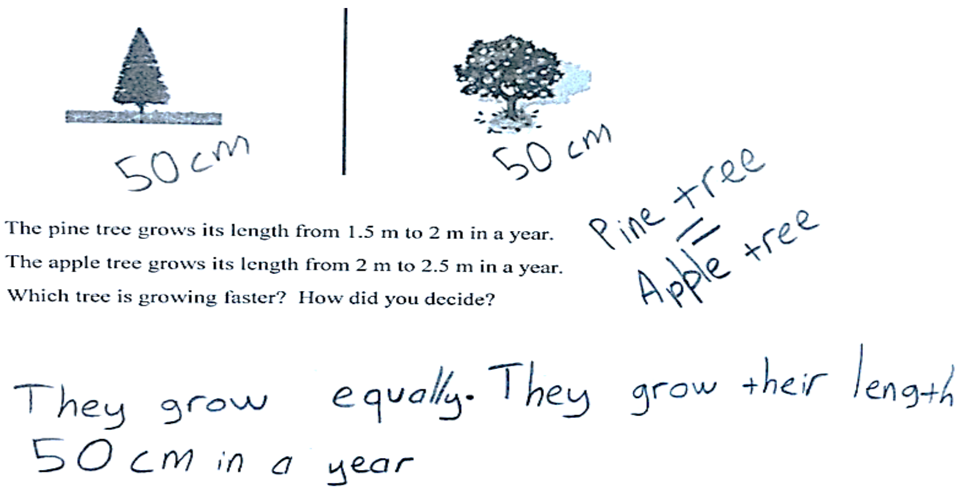


Figure 3. The student answer to the 6th question

The question was prepared in compliance with the learning outcome; “Student calculates one quantity as a percentage of another quantity.” T5’s student’s answer was directed to the teacher and s/he was asked “What could your student be thinking while giving this answer?”. T5’s answer was:

> T5: “The student could not proportion, so s/he paid attention to the difference between two trees in length and s/he gave a wrong answer to the question.”

T5 considered the student’s answer as a mistake while it was clear that the student had the misconception in the literature: “The increase or decrease in an amount is same as increase or decrease in percentage”. For the same question, T19 interpreted his student’s answer which was similar to the T5’s student as below:

> T19: “My student had a misconception thinking that the differences between two trees in length are equal, which made him think that two trees got longer equally.”

This statement apparently shows that the teacher completely understood his/her student’s point of view. 50% of the interviewed teachers could interpret the reason of the students’ misconceptions while 50% of them could not exactly make their interpretation and thought that they made a mistake. Figure 4 shows the written answer given by a student of T5 to the 10th question in the test.

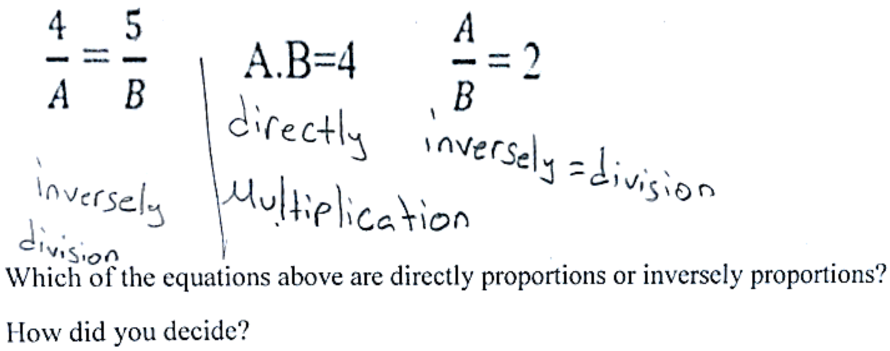


Figure 4. The student answer to the 10th question

The question aims at the learning outcome; “Students explain whether the two variables are directly proportion and inversely proportional.” T5’s reaction to his/her students’ answer in the misconception test when asked “What could your student think while giving this answer?” was as follows:

> T5: “The student could not do it correctly because she/he could not do abstract reasoning. The student could have given value; I solved a lot of similar questions.”

It is clearly seen that the teacher could not understand the student’s point of view, whereas the student had the misconception in the literature:” At the direct proportion variables are multiplied and at the inversely proportion the variables are divided.” and had the generalized misconception that multiplication was done in direct proportion and division was done in inverse proportion. Of the interviewed teachers, 30% were able to interpret the reason of the students’ misconceptions while 70% of them could not exactly interpret them, and thought that they made a mistake.

4. Discussion and Conclusions

As a result of the study, teachers’ viewpoints regarding the misconceptions were found narrow, and their skills of noticing students’ misconceptions were low. When it is considered that the determination of students’ misconceptions about a concept is among the studies that should be carried out within the scope of measurement and evaluation, it can be said that the pedagogical content knowledge adequacy, which includes the ability to reveal the students' point of view and to treat misconceptions by analyzing these thoughts highlighted by Shulman (1986) is low. In other words, this result shows that teachers’ ability to notice misconceptions revealed by Jacobs et al. (2010) is low in the context of “Commenting on students’ mathematical intellection”. The studies also show that teachers and prospective teachers are inadequate in interpreting students' thoughts (Boz, 2004) and that their ability to discern the misconceptions in the sample case video recordings is low (Estapa et al., 2018; Goldsmith & Seago, 2011; Jacobs et al., 2010; Özdemir-Baki & Işık, 2018; van Es 2011). In these studies, the videos were watched and all the components of noticing were discussed by interpreting the course process. It was seen that the teachers were more focused on teachers than students' understanding in video recordings, so this approach is limited in terms of in-depth analysis of student thinking (Tataroğlu-Taşdan, 2019; van Es & Sherin, 2008; Ulusoy & Çakıroğlu, 2018). In this study, unlike the literature, "interpreting students' mathematical understanding" was discussed in detail and it was seen that teachers' ability to recognize their own students in terms of written answers is low.

Although the written responses of the students were addressed and the teachers were asked to think directly on the answers of their own students in order to focus on distinguishing the awareness of the other components that appeared in the videos and only to notice the students' understanding, the teachers were still focused on the correct or wrong answer rather than the student's understanding. They did not notice misconceptions. According to Boz (2004) analyzing an answer starts with determining whether the student's answer is true or false. The next steps are to detect the error, to express the sources of the error and to plan how to prevent the error. In this study, it is seen that teachers cannot continue the process. Goldsmith and Seago (2011) found that if the written answers belong to the students they are familiar with, the teachers make comments about the students' understanding considering the situation of the student in the classroom. However, in this study, it was seen that the teachers focused on the answer right away and that they did not think about their situation in the classroom when interpreting the answers of their students. This shows that more studies are needed to explain the meaning of the noticing skills to students' misconceptions.

It is clear that only teachers with the ability to notice misconceptions can determine the meaningful mathematical thoughts of their students and make decisions for the next instructional step depending on the students' thoughts and understandings (Krupa, Huey, Lesseig, Casey, & Monson, 2017). Teachers, who do not have the ability to notice, manage the process without the opportunity to examine the knowledge construction in students' mental processes. In this context, revealing the teachers' noticing skills in the context of interpreting mathematical understanding of students, this study forms a basis for the studies to develop these skills.

This study has some limitations. The most important of these is that the noticing skills of teachers are limited by the misconceptions in the test developed by the researchers. In future studies, misconceptions can be diversified and it can be examined whether noticing skills have changed or not. Another limitation is that the noticing skills of teachers are evaluated in two dimensions as *notice the misconceptions* and *do not notice the* *misconceptions*. In line with the results obtained from this study, studies can be conducted to investigate teachers' skills of noticing misconceptions from written responses according to the levels. It is clear that there is a need for researches to increase teachers' noticing of misconceptions. In-service training can be given to teachers on how to evaluate students' written responses. With these trainings, awareness of misconceptions can be developed. These practices are required to be on an ongoing basis. It can be recommended that the deficiencies should be detected and resolved through collaboration with the Ministry of Education and the universities in Turkey.

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